

SCALE-DEPENDENT SLIP AT POROUS INTERFACES: MICRO-PIV MEASUREMENTS OF BEAVERS–JOSEPH AND SAFFMAN COEFFICIENTS

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ABSTRACT

The interaction between a free-flow and a porous medium is frequently modeled using interfacial slip conditions, such as the Beavers–Joseph formulation. Despite extensive theoretical and numerical studies, the experimental determination of slip coefficients and their dependence on pore-scale geometry and averaging procedures remains insufficiently constrained. In this work, flow coupling at a fluid–porous interface is investigated using micro-particle image velocimetry (μ -PIV) measurements in Hele-Shaw-type microfluidic models. Both ordered and disordered porous media are examined, with bulk porosities of 55%, 75%, and 85%. Pore-scale velocity fields are systematically upscaled via spatial averaging to the representative elementary volume (REV) scale, enabling direct quantification of slip velocities, interfacial velocity gradients, and the corresponding slip coefficients. The interfacial slip velocity is governed primarily by local pore geometry, whereas the Darcy velocity within the porous matrix depends strongly on bulk porosity and structural disorder. Both the Beavers–Joseph and Saffman slip coefficients vary not only with porosity but also with the size and shape of the averaging filter. At local and sub-REV scales, the two coefficients can differ by more than 100%, with convergence observed only after REV-scale averaging in low-permeability systems. In highly permeable ordered media, insufficient averaging can even yield non-physical negative slip coefficients. These findings show that slip coefficients are scale-dependent parameters rather than intrinsic material properties. Robust estimation therefore requires consistent REV-scale averaging and explicit reporting of the averaging procedure. More broadly, the results motivate standardized, geometry-aware protocols for determining and comparing interfacial slip conditions in porous media.

KEYWORDS

Free-flow/porous media coupling, Fluid-porous interface, Slip velocity, Representative elementary volume scale, REV scale, Micro-Particle Image Velocimetry, Micro-PIV, Beavers–Joseph condition, Saffman condition

WHY THIS PAPER MATTERS

Predicting flow at fluid–porous interfaces remains challenging due to the scale dependence of interfacial slip parameters. Experiments across ordered and disordered porous structures demonstrate that slip coefficients depend on both pore-scale geometry and spatial averaging. These results establish the need for consistent averaging procedures to ensure reliable characterization and comparison of interfacial flow.



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1. INTRODUCTION

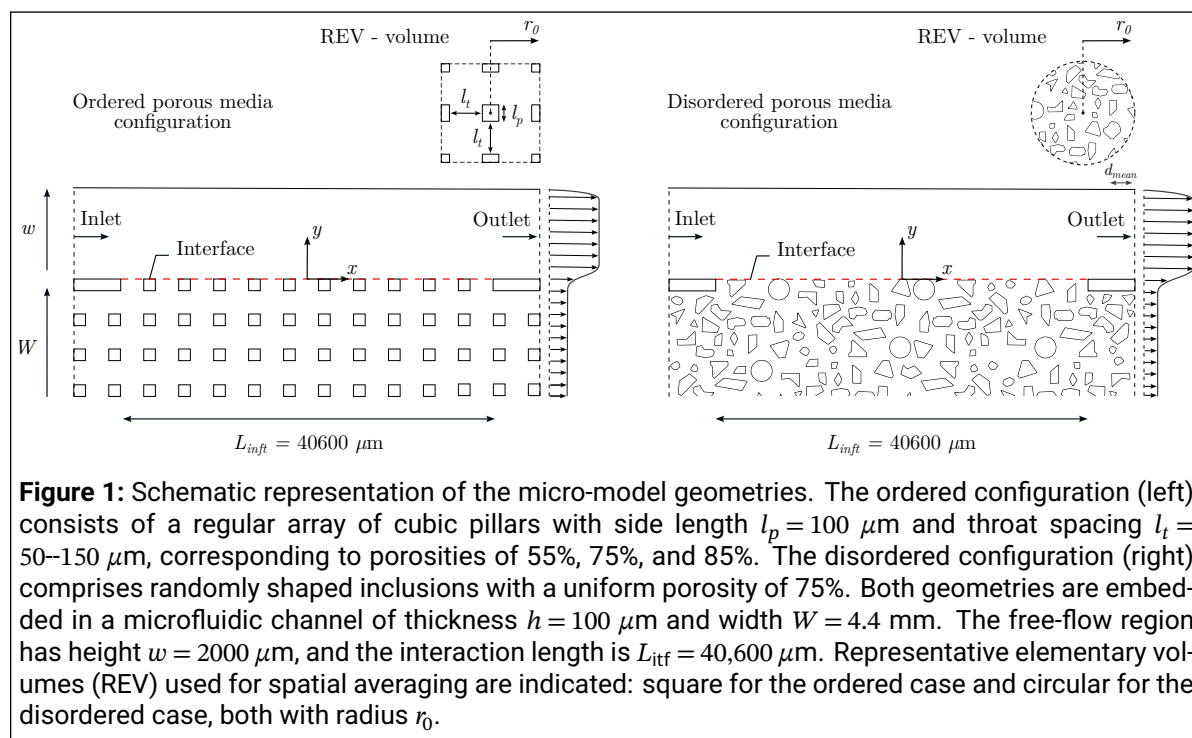
Coupled systems describing interactions between a free flow and a porous medium have been studied extensively for more than half a century, motivated by a broad range of technological applications (1, 11, 18, 24, 25, 35). A central challenge is the formulation of macroscopic interfacial boundary conditions that accurately capture the coupling between the two domains (8).

Achieving a clear and quantitative characterization of interfacial mechanisms across multiple scales remains inherently challenging. From a numerical standpoint, directly solving the Navier–Stokes equations over the full domain while fully resolving the porous microstructure is computationally prohibitive (9). Moreover, in the absence of detailed geometric reconstructions of internal pore networks, numerical studies commonly resort to simplified configurations, employing idealized porous matrices with prescribed geometries (6, 7, 12, 17, 19, 37, 42). While useful, such idealizations can obscure key pore-scale mechanisms that govern macroscopic transport.

The representative elementary volume (REV) formulation offers an effective means to bypass the direct resolution of pore-scale flow by homogenizing microscopic details through volume-averaging techniques (27, 29). On this scale, flow through porous media is commonly described by Darcy’s law (13) or its extensions that account for inertial and viscous effects. Despite this simplification, accurately solving the flow problem remains challenging.

Historically, two main modeling strategies have been proposed, distinguished by the governing equations employed. The first, known as the two-domain approach, couples the Stokes equations in the laminar free-flow region with the Darcy equation in the porous medium. This coupling, however, requires specific interface conditions, as the two models are of different orders and Darcy’s law neglects viscous diffusion, thus preventing velocity continuity across the interface. However, it is worth noting that recent studies (40, 41) have shown that a variable-permeability formulation can overcome this limitation, allowing boundary layers to develop within the Darcy framework. Within this context, the coupling is achieved through interface conditions, and several formulations have been proposed. The earliest and most widely known being the Beavers and Joseph slip condition (3) introduced in 1967. This semi-empirical boundary condition introduces a velocity discontinuity at the interface controlled by the parameter α , also known as the Beavers–Joseph slip coefficient. Subsequent analytical work by Saffman (33) confirmed its theoretical validity under low-permeability conditions ($k \rightarrow 0$), while later studies demonstrated that the slip coefficient depends primarily on the interfacial geometry (4, 32, 36) and its location (22, 23, 34), rather than on fluid properties such as viscosity (14).

A second modeling approach employs a single governing equation, typically the Brinkman equation, over the entire domain, with parameters such as permeability and porosity varying continuously across a transition region of finite thickness δ . This method requires prior knowledge of the spatial variation of effective properties, which is rarely available for complex porous materials. In



addition, the definition of an effective viscosity remains an open issue, with no consensus on its relationship to the actual viscosity, as some studies report a ratio greater than unity, others lower, and some assume equality (2, 21, 22, 23, 27). Finally, the validity of the Brinkman correction itself, as a modification of Darcy's law, remains debated (8, 9, 10).

Hybrid approaches have also been proposed, such as coupling the Stokes and Brinkman equations, as in the work of Neal and Nader (26), which established a direct correlation between the square root of the effective viscosity and the Beavers–Joseph slip coefficient. In the 1990s, Ochoa-Tapia and Whitaker (27, 28, 29) introduced a new approach based on the volume averaged transport equations applied to the entire domain. They proposed a jump condition in the stress field at the interface while preserving continuity of other quantities such as velocity. This formulation introduced a new dimensionless parameter β , which, despite rigorous derivation, could not be explicitly determined due to the unresolved closure problem. Nonetheless, this work opened the perspective that an interface devoid of physical thickness and thermodynamic properties might be an inadequate representation for modeling such systems. Furthermore, more advanced and innovative numerical approaches have been developed over the last decade. These methods typically rely on the Lattice Boltzmann Method (LBM) coupled with more sophisticated strategies for representing the interaction with the porous domain. For example, in the study by Kang et al (20), the LBM framework is employed to model the coupled free-flow/porous-medium system through a decomposition into multiple regions. Specifically, the computational domain is divided into a clean channel region, an inner Brinkman layer, an outer Brinkman layer, and a bulk porous region. Operating at the mesoscopic scale, this approach introduces an additional level of domain decomposition in order to capture more accurately the variations in momentum transfer occurring within the interfacial region between the free-flow and porous-media domains. While this methodology provides a more detailed description of the underlying transport mechanisms, it does so at the expense of increased model complexity and computational cost. As such, it can be regarded as an extension of the aforementioned hybrid framework, aimed at improving the representation of interfacial phenomena.

Despite these extensive theoretical and numerical efforts, direct experimental investigations that resolve the pore-scale flow field and quantitatively assess the role of porous-domain geometry at

the free-flow interface remain scarce (37). Building on previous experimental studies on velocity-field reconstruction in coupled free-flow and porous-media systems (14, 15, 16), we present an experimental investigation of the interaction between a free-flow region and a disordered porous medium, with emphasis on pore-scale geometry. The velocity field within disordered porous systems is fully reconstructed and directly compared with an ordered reference configuration, allowing the effects of porous-domain geometry at the fluid-porous interface to be isolated. The study further investigates the effects of the averaging procedure, specifically the influence of the kernel structure and filter size, on the determination of the Beavers–Joseph and Saffman slip coefficients. It is shown that the values of these parameters depend on the specific averaging procedure adopted to transition from a pore-scale description to a mesoscopic representation of the flow. This result highlights a fundamental limitation of the volume-averaging approach as it is currently applied. In the absence of a standardized averaging framework, comparisons between experimental and numerical studies may therefore lead to inconsistent or ambiguous interpretations.

2. MATERIALS AND METHODS

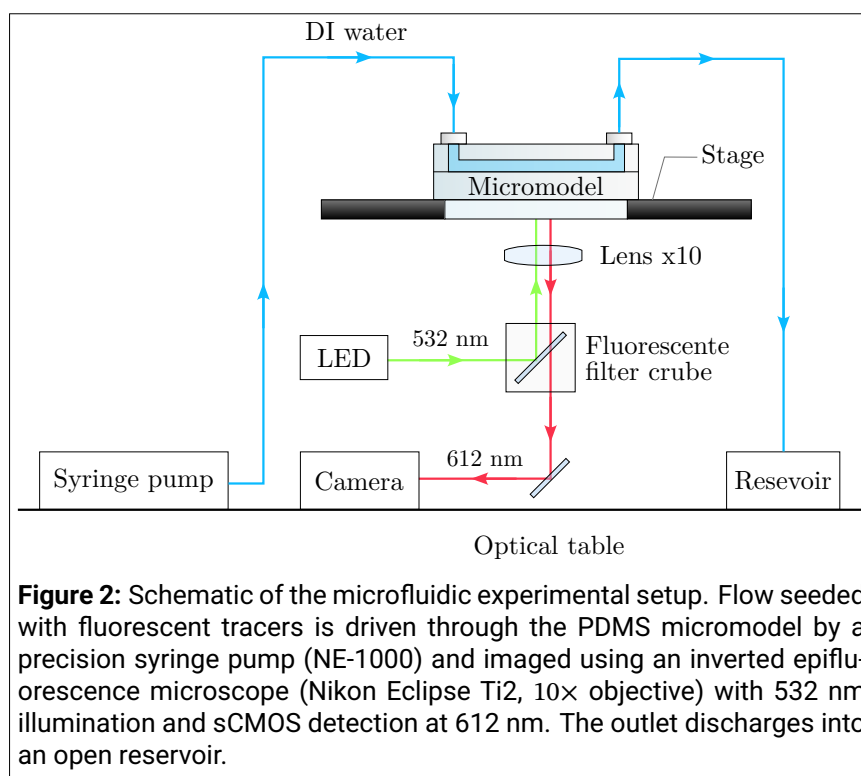
2.1. Micro-models design and fabrication

The micro-models employed in this study comprise two distinct porous configurations, representing ordered and disordered porous media, respectively. Both structures are investigated in order to systematically assess the influence of pore-scale geometry on the interaction between a free-flow region and a porous domain.

From a geometrical perspective, each micro-model comprises a free-flow region devoid of solid obstacles together with a porous domain populated by rigid inclusions that introduce additional hydrodynamic resistance. The porous medium is integrated within a microfluidic device equipped with multiple inlet and outlet ports, enabling independent access to the free-flow and porous regions during the initial saturation stage. During the experiments, fluid circulation is restricted to the free-flow inlet and outlet, while the ports connected to the porous domain are sealed after saturation. During saturation, care is taken to prevent air entrapment, particularly in the disordered configurations. Since water is used as the working fluid and the structures are fabricated from hydrophobic Polydimethylsiloxane (PDMS), the micro-models are pre-conditioned with CO₂ to reduce solid-liquid interfacial tension and ensure uniform pore-space saturation. A 100 μm -thick splitter separates the free-flow and porous regions over approximately 27 mm at both the inlet and outlet, ensuring a fully developed free stream before interaction. The two regions then interface directly over a length of $L_{\text{iff}} = 40,600 \mu\text{m}$.

The ordered porous configuration consists of a regular array of cubic pillars with a side length of $l_p = 100 \mu\text{m}$. The bulk porosity of the medium is controlled by a single geometric parameter, namely the throat length l_t , defined as the spacing between adjacent pillars. By varying l_t between 50 μm and 150 μm , three target porosities of approximately 55%, 75%, and 85% are obtained. The width of the porous cavity is kept constant at $W = 4.4 \text{ mm}$ for all configurations.

The disordered porous medium is constructed from a base unit cell of randomly shaped polygons with a bulk porosity of 75%, replicated across the domain using isometric transformations to maintain statistical disorder without introducing periodicity. **Figure 1** illustrates the porous geometries, the REV, and the reference coordinate system, whose origin is located at the free-flow/porous interface. The free-flow region is designed in a Hele–Shaw configuration, with a height of $w = 2000 \mu\text{m}$ and a thickness of $h \approx 100 \mu\text{m}$, resulting in an aspect ratio of approximately 20. This strong confinement in the out-of-plane direction promotes a quasi-two-dimensional flow field and ensures a well-defined and reproducible interaction with the porous medium.



The micro-models are fabricated using standard photolithography and soft-lithography techniques. A master mold defining the porous geometry is produced via ultraviolet photolithography on a silicon wafer using a CAD-designed photomask. Polydimethylsiloxane is cast onto the mold, cured, and demolded to obtain the structured layer, which is subsequently bonded to a flat PDMS slab and a glass substrate via oxygen plasma treatment. Inlet and outlet ports are created by precision punching, and additional PDMS gaskets are employed to enhance sealing and mechanical robustness during operation.

2.2. Experimental setup

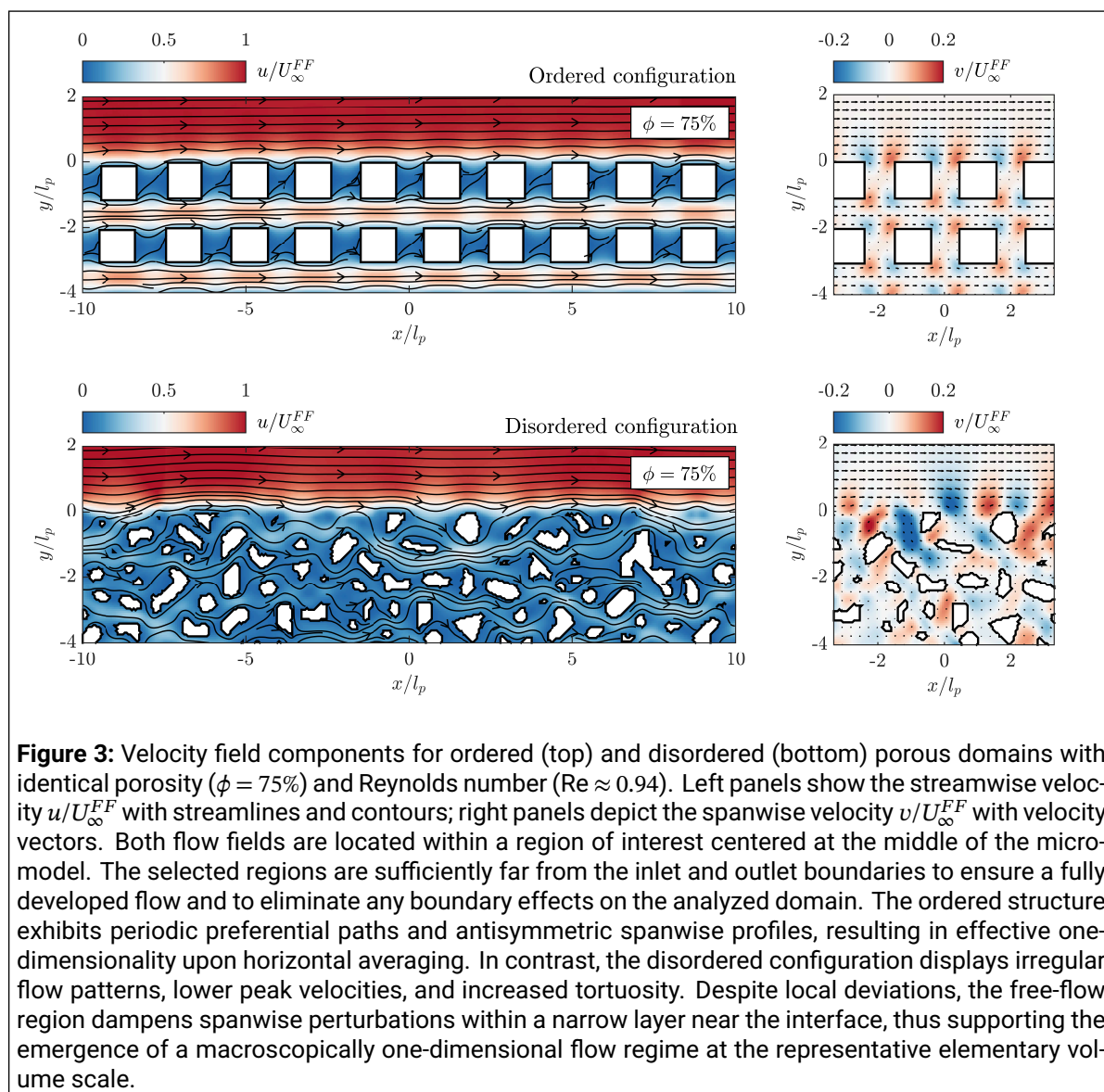
Flow measurements were conducted in a custom microfluidic setup integrating a PDMS micro-model, precision NE-1000 single-channel pump, and optical diagnostics system, as reported by **Figure 2**. The device was mounted on a vibration-isolated optical table and imaged using an inverted epifluorescence microscope^a coupled with a high-sensitivity sCMOS camera^b.

Tracer particles were selected to minimize sedimentation while optimizing the signal-to-noise ratio for compatibility with the camera sensor. Their size ensured that each particle occupied approximately 2–5 pixels on the recorded image, enabling accurate sub-pixel displacement detection with minimal tracking error (31). Flow rates of 10, 30, and 60 $\mu\text{L}/\text{min}$ were imposed in both configurations, corresponding to Reynolds numbers of 0.16, 0.47, and 0.94, respectively, thereby ensuring creeping flow conditions. Image acquisition parameters were adjusted according to the local flow velocities, with frame rates spanning from 333 to 2300 fps.

Velocity fields were extracted via micro-particle image velocimetry (μPIV) using a multi-pass cross-correlation algorithm based on fast Fourier transforms (FFT) with iterative window deformation. Subpixel-scale displacement vectors were computed by fitting a 2×3 -point Gaussian function to the correlation peak (38, 39). The final interrogation step employed $16 \times 16 \text{ px}^2$ windows with 50% over-

^aNikon Eclipse Ti2

^bKinetix, Photometrics



lap, yielding velocity fields with a spatial resolution of approximately $5 \mu\text{m}$ per vector—adequate for resolving interstitial flow within the porous domain.

This approach enabled high-fidelity quantification of flow structures in both free and confined regions of the domain. Additional information on the experimental apparatus and procedures can be found in prior studies (14, 15, 16).

3. MICRO-PIV MEASUREMENTS AND 2D SURFACE CONTOURS

Figure 3 illustrates the measured velocity fields for two representative cases of porous domains, one with an ordered geometry and the other with a disordered configuration. The section shown represents a region of interest (ROI) of approximately 2 mm , located at the center of the micro-model, at a distance of about $20 \times w$ from both the inlet and outlet boundaries. This positioning is intended to avoid any boundary effects and to ensure that the flow is fully developed before reaching the analyzed region. The streamwise velocity component is shown on the left side of the figure and represents the dominant direction of the flow within the domain.

The top-left panel illustrates the spatial distribution of the streamwise velocity component for the ordered porous configuration, whereas the bottom-left panel reports the corresponding distribu-

tion for the disordered counterpart. Both velocity fields were acquired under conditions of equal bulk porosity, i.e. of $\phi = 75\%$, and at a Reynolds number equal to 0.94. The right side of the figure reports the spanwise velocity component for the same configurations. All velocity fields are normalized by the characteristic velocity U_{∞}^{FF} , defined in each case as the undisturbed streamwise component of the bulk Darcy velocity in the free-flow domain.

The streamwise velocity field within the ordered porous domain exhibits a periodic and well-organized structure. Regions of elevated velocity, hereafter referred to as preferential flow paths, develop within the void spaces between vertically aligned rows of cubic pillars. Along these paths, the flow alternates between local acceleration and deceleration as a consequence of incompressibility and variations in the available cross-sectional area. At the interface ($y/l_p = 0$), an external boundary layer develops above the porous domain. This layer remains confined within approximately $200\ \mu\text{m}$ from the pillar tips. Beyond this distance, the boundary layer is fully developed, and the velocity profile approaches a uniform distribution corresponding to the Darcy velocity in the free-flow region. This plug-flow regime results from the geometrical constraints of the micro-model architecture. The free-flow region behaves as a Hele-Shaw cell, with a parabolic velocity profile in the depth-wise (z) direction due to strong out-of-plane confinement. Accordingly, the depth-averaged velocity in the free-flow region follows the classical Darcy expression, consistent with theoretical predictions (16).

Similar flow dynamics are observed at the fluid-porous interface. A boundary layer forms at $y/l_p = 0$ and extends from the porous matrix into the free-flow region. Its formation is driven by alternating slip and no-slip conditions imposed by the irregular interfacial geometry. The layer thickness is comparable to that of the ordered case, remaining confined within approximately $200\ \mu\text{m}$ from the pillar tips, but its internal structure is more heterogeneous. Beyond this transition region, the free-flow region recovers a plug-like velocity profile, consistent with the ordered configuration. Within the disordered porous matrix, preferential flow pathways are still present. These pathways coincide with regions of locally reduced solid fraction, where fluid acceleration produces streamwise velocity peaks of up to approximately 50% of the free-flow Darcy velocity. This contrasts with the ordered configuration, in which velocity maxima along preferential paths reach values of about 80% of the free-flow Darcy velocity. Despite identical bulk porosity, the disordered geometry induces a stronger attenuation of the mean flow, highlighting the role of structural organization in controlling hydrodynamic transport. This behavior is primarily attributed to the higher tortuosity of the disordered medium, which forces the flow to deviate from straight streamwise trajectories. In the disordered case, streamlines undergo frequent directional changes as the fluid follows paths of least resistance, in contrast to the predominantly straight trajectories observed in the ordered configuration. These large angular deviations, occasionally exceeding 70° , redistribute kinetic energy from the streamwise to transverse components, leading to a reduction of the local streamwise velocity. This behavior directly impacts the macroscopic flow response, with local tortuosity controlling permeability even when bulk properties are identical.

In the ordered configuration, the spanwise velocity component exhibits a periodic distribution that is antisymmetric about the center of the gap between adjacent pillars. As a consequence, averaging along the x -direction yields a zero mean spanwise velocity at all vertical positions. Despite local spanwise velocities reaching up to 20% of the free-flow value, the flow is therefore effectively one-dimensional at the REV scale.

In the disordered configuration, the symmetry properties observed in the ordered case are not preserved. Although the spanwise velocity component still attains local peaks of up to 20% of the bulk free-flow velocity, its spatial distribution lacks periodicity and antisymmetry, so horizontal averaging does not inherently yield a vanishing mean. However, the interaction between the porous matrix and the external free-flow region is confined to a narrow zone extending only a few hun-

dred microns from the interface, which strongly limits the impact of these local deviations. Fluid ejected locally from the porous matrix is rapidly redirected back within distances comparable to the pore spacing, typically of order $50\ \mu\text{m}$ for an average pore diameter of $20\ \mu\text{m}$. As a result, streamline patterns remain nearly aligned across the interface despite local perturbations. Consistently, line-averaged velocity profiles indicate that the spanwise velocity component remains below 1% of the bulk free-flow value at all spanwise positions, confirming that the flow remains effectively one-dimensional at the REV scale. Finally, within the porous bulk, the stochastic geometry promotes momentum redistribution and suppresses transverse velocity components, so that streamwise transport dominates and a Darcy-like mean velocity profile emerges at the macroscopic scale.

4. SLIP COEFFICIENTS AND REV SCALE

4.1. Superficial averaging

Porous materials are characterized by complex and often poorly known internal structures. This complexity is closely associated with the challenge of identifying the relevant range of length scales governing both the porous microstructure and the corresponding flow dynamics. A mesoscopic description of flow through a porous medium begins with the definition of the superficial volume average of a generic quantity ψ . This average, denoted by $\langle\psi\rangle^s$, is rigorously defined (27) by **Equation 1**:

$$\langle\psi_\beta\rangle_{\mathbf{x}}^s = \frac{1}{V} \int_{V_\beta(\mathbf{x})} \psi_\beta(\mathbf{x} + \mathbf{y}_\beta) dV_{\mathbf{y}} \quad (1)$$

The average is evaluated at the macroscopic position $\mathbf{x}$, corresponding to the centroid of the averaging volume. The total volume V denotes the REV, which includes both the fluid and solid phases. In a more general form, **Equation 1** can be written compactly as:

$$\langle\psi\rangle = \int_V m(\mathbf{y}) \gamma(\mathbf{r}) \psi(\mathbf{r}) dV = \overline{m} * [\gamma\psi] \quad (2)$$

This alternative formulation (5, 30) introduces the phase-indicator function $\gamma(\mathbf{r})$ and the weighting function $m(\mathbf{y})$. The vector $\mathbf{r}$ denotes the position of a point within the averaging volume V , with $\mathbf{y} = \mathbf{r} - \mathbf{x}$. The function $\gamma(\mathbf{r})$ equals unity when $\mathbf{r}$ lies within the fluid phase and vanishes otherwise. The function $m(\mathbf{y})$ is a smooth, non-negative weighting kernel that determines the spatial contribution of neighboring points in the averaging procedure. In this formulation, the integration is extended over the entire space $\mathbb{R}^3$, and the kernel $\overline{m}$ is defined as $\overline{m}(\mathbf{x} - \mathbf{r}) \equiv m(-\mathbf{y})$.

4.2. Weighting functions

As shown in Equation 2, the accuracy of the averaging procedure critically depends on the choice of the weighting function. Although any smooth function may be used in principle, the selection of $m(\mathbf{y})$ should reflect the structural organization of the porous medium (30). Accordingly, the filter volume must be consistent with the topology of the domain to ensure physically meaningful averages. In three-dimensional systems, spherical volumes are generally appropriate for disordered media, whereas rectangular volumes are better suited to ordered or periodic structures. The choice of filter size further involves a trade-off: the averaging radius r_0 must be large enough to smooth the field at the Darcy scale, yet sufficiently small to retain information from the underlying microscopic field at the interface (5).

In this work, ordered porous domains are analyzed using a pyramidal cellular filter, specifically a bilinear hat weighting function. Here, $\mathbf{y} = (y_x, y_y)$ denotes the local coordinate relative to the

centroid of the averaging volume, and r_0 defines the half-length of the square filter support along both spatial directions. The corresponding weighting function $m(\mathbf{y})$ is defined by **Equation 3**:

$$m(\mathbf{y}) = \begin{cases} \frac{(r_0 - |y_x|)(r_0 - |y_y|)}{r_0^4}, & \text{if } |y_x| \leq r_0, |y_y| \leq r_0, \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

For domains with a randomly distributed solid phase, a circular top-hat filter is employed. Here, r_0 denotes the radius of the averaging disk. The filter assigns equal weight to all points within a circular support of radius r_0 and zero weight elsewhere. The corresponding two-dimensional weighting function is given by **Equation 4**:

$$m(\mathbf{y}) = \begin{cases} \frac{1}{\pi r_0^2}, & \text{if } \|\mathbf{y}\| \leq r_0, \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

As discussed in previous studies (5, 30), the choice of an appropriate filter size for averaging in disordered porous media must satisfy specific scale-separation conditions to ensure both spatial representativeness and continuity of the averaged fields. These conditions can be expressed as follows:

$$d_{\text{mean}} \ll r_0 \ll L_u \quad (5)$$

$$r_0^2 \ll L_\phi L_u \quad (6)$$

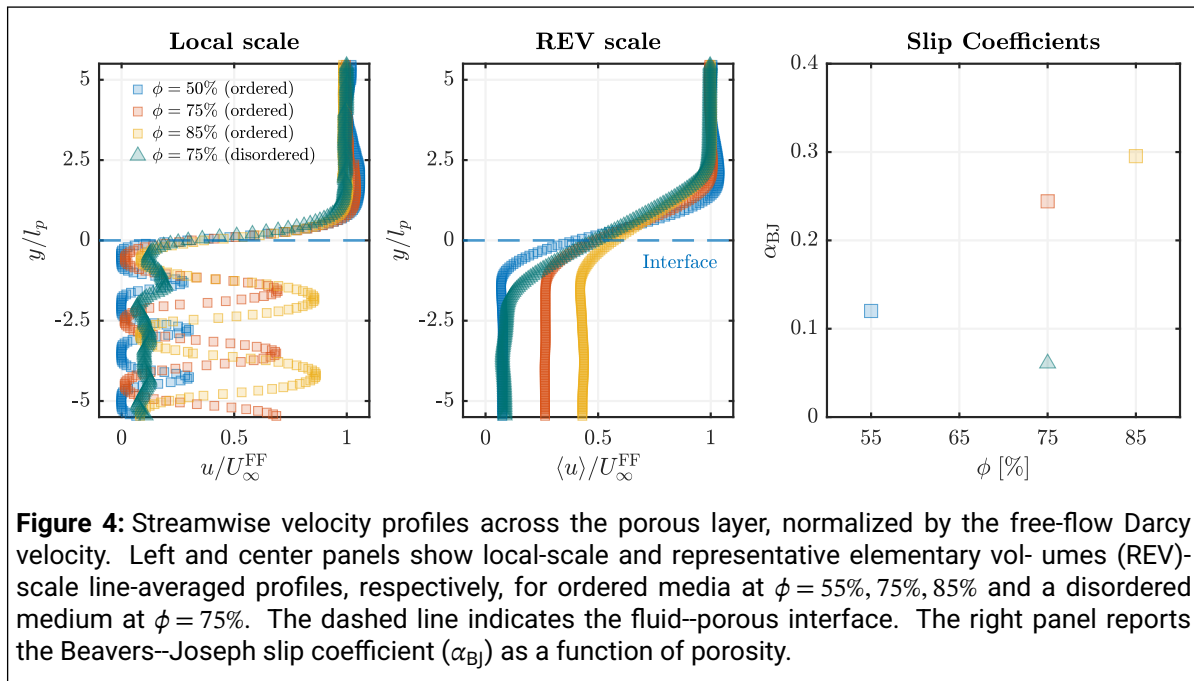
$$r_0^2 \ll L_\phi L_p \quad (7)$$

Here, d_{mean} denotes the mean pore diameter, L_u the characteristic length scale of variations in the volume-averaged velocity field, L_ϕ the correlation length of porosity fluctuations, and L_p the characteristic scale of pressure gradients. Satisfaction of these inequalities ensures that the averaging procedure retains essential microscale information while enabling a consistent macroscopic description.

The analysis of the systems considered in this study indicates that, in ordered porous domains, the optimal averaging radius r_0 follows a well-defined geometric scaling. In particular, r_0 scales with the sum of the characteristic geometric lengths, such that $r_0 \propto l_t + l_p$. Accordingly, for porosities of 55%, 75%, and 85%, the corresponding optimal values of r_0 are 150 μm , 200 μm , and 250 μm , respectively. For the disordered porous domain, the optimal averaging radius is $r_0 = 201 \mu\text{m}$, satisfying the scale-separation conditions given in **Equations 5–7**.

4.3. Mesoscopic-Scale Flow Profiles

Figure 4 compares the streamwise velocity profiles, normalized by the Darcy velocity in the free-flow region, obtained by line averaging of the velocity field. Local-scale profiles are shown in the left panel, while the corresponding mesoscopic (REV-scale) profiles are reported in the central panel. All reported profiles correspond to $\text{Re} = 0.94$ (Stokes-flow regime), for which the normalized velocity profiles are Reynolds-number independent and collapse onto a single distribution, making the results representative of all cases considered (15).



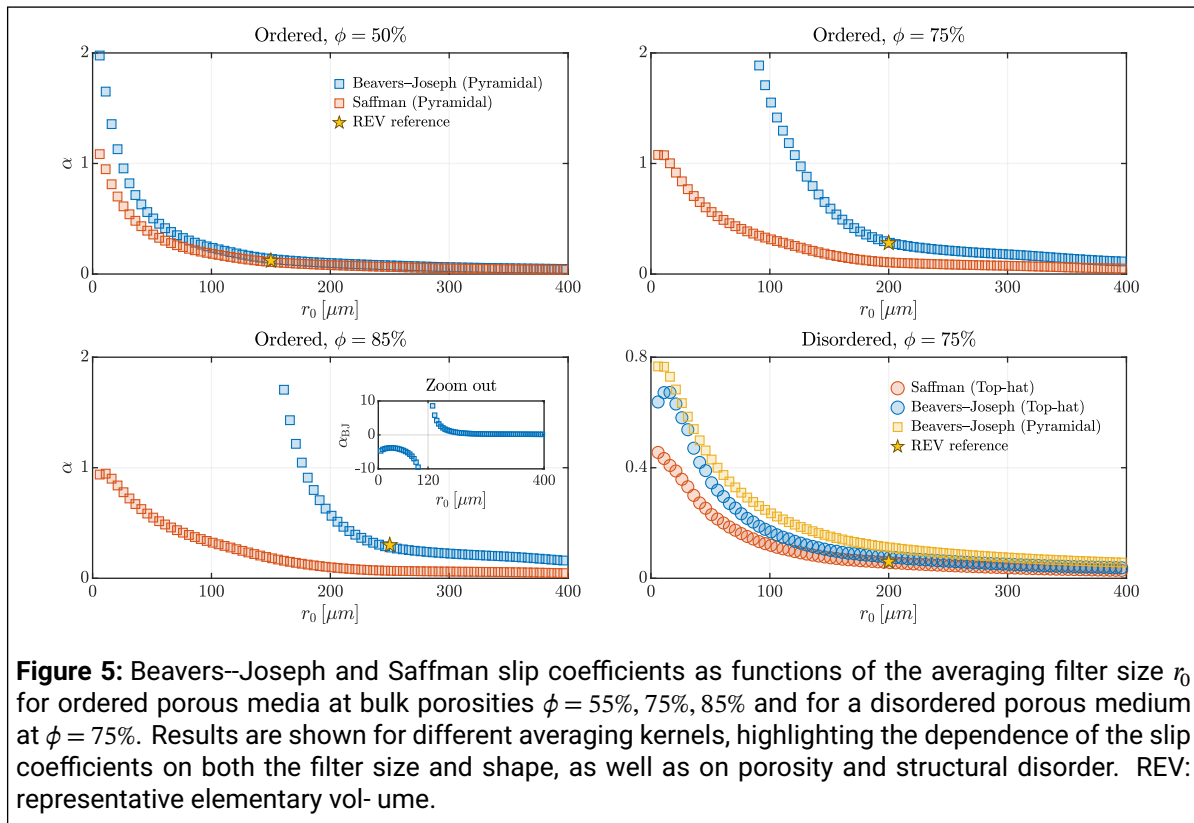
At the local scale, the dimensionless slip velocity at the interface increases with bulk porosity in ordered porous media, reflecting the non-linear effect of the increasing characteristic spacing l_t between adjacent inclusions. Specifically, the slip velocity reaches 29%, 31%, and 36% of the free-flow Darcy velocity for porosities of 55%, 75%, and 85%, respectively, while a value of 29.6% is observed for the disordered medium. A similar trend is found at the REV scale, where the slip velocity in ordered systems increases to 40%, 49%, and 57% for the same porosities, and reaches 48.8% in the disordered case. For a given bulk porosity, the slip velocity remains comparable between ordered and disordered configurations, consistent with their similar mean inter-inclusion spacing l_t .

In contrast, the Darcy velocity within the porous domain depends strongly on both porosity and structural disorder. In ordered systems, the normalized Darcy velocity increases monotonically with porosity, reaching 8%, 26%, and 43% for porosities of 55%, 75%, and 85%, respectively. At 75% porosity, the disordered system exhibits a much lower normalized Darcy velocity of 9.2%, comparable to that of the ordered system at 55%. These results indicate that the slip velocity at the interface is mainly governed by the local interfacial porosity, whereas, conversely, the flow within the porous bulk is strongly affected by both porosity and structural disorder.

Finally, we consider the uniformity of the local-scale velocity distribution in the deeper layers of the porous domain, as shown in the left panel. In ordered configurations, well-defined preferential pathways develop with a periodic spatial organization, giving rise to alternating regions of high and low momentum and, consequently, to a strongly heterogeneous velocity field. In contrast, the disordered geometry promotes a more uniform redistribution of the flow across the porous domain. Thus, while ordered pore structures lead to heterogeneous line-averaged velocity distributions, disordered configurations result in a more homogeneous velocity field within the porous medium.

Finally, the rightmost panel reports the slip coefficient at the REV scale as originally defined by Beavers and Joseph (α_{BJ}) and given in **Equation 8**:

$$\alpha_{BJ} = \frac{\sqrt{k_{PM}}}{u_s - U_{\infty}^{PM}} \left. \frac{\partial u}{\partial y} \right|_{y=0} \quad (8)$$



Here, u_s denotes the interfacial slip velocity, U_∞^{PM} the Darcy velocity within the porous medium, and k_{PM} the permeability of the porous matrix. The slip coefficient increases with bulk porosity, displaying an approximately linear dependence on the bulk porosity. Specifically, values of $\alpha_{BJ} = 0.12$, 0.24 , and 0.30 are obtained for porosities of $\phi = 55\%$, 75% , and 85% , respectively. It is worth noting that, although lower porosities in ordered systems are associated with larger interfacial velocity gradients, the term $\sqrt{k_{PM}}/(u_s - U_\infty^{PM})$ decreases more rapidly as porosity decreases. This reduction offsets the increase in the velocity gradient, leading to an overall decrease of the slip coefficient at lower porosities.

These results show that α_{BJ} depends not only on bulk porosity but also on the interfacial pore geometry and structural disorder. At identical porosity ($\phi = 75\%$), the disordered configuration exhibits a much lower slip coefficient, $\alpha_{BJ} = 0.06$, approximately one quarter of the value in the ordered case. Because interfacial velocity gradients and slip velocities are comparable, this reduction is attributed to the lower permeability of the disordered porous medium.

4.4. Scale dependence of interfacial slip coefficients

The final aspect examined concerns the dependence of the Beavers–Joseph slip coefficient on the averaging procedure, including the kernel shape and its characteristic length scale. **Figure 5** reports the slip coefficients obtained for the four configurations considered. For reference, the figure also includes the Saffman slip coefficient α_S , defined (33) in **Equation 9** and derived in the asymptotic limit of vanishing permeability ($\kappa \rightarrow 0$).

$$\alpha_S = \frac{\sqrt{k_{PM}}}{u_s} \left. \frac{\partial u}{\partial y} \right|_{y=0} \quad (9)$$

Considering the first three panels corresponding to ordered porous media, for which a pyramidal averaging filter is consistently employed, both the Beavers–Joseph and Saffman slip coefficients

are found to depend on the size of the averaging filter used to reach the REV scale. This result is particularly relevant, as many experimental studies extract slip coefficients from line-averaged PIV velocity fields without explicitly reporting either the averaging procedure or the characteristic kernel size, and sometimes derive these values directly at the local scale (37). Beyond this scale dependence, the separation between the two slip coefficients increases with the permeability of the porous medium. For the lowest permeability case ($\phi = 55\%$) the two curves nearly coincide; however, when evaluated from local-scale the Saffman coefficient differs from the Beavers–Joseph coefficient by approximately 100%. This discrepancy increases rapidly with increasing porosity.

A fundamental limitation is observed for the highly permeable ordered medium at $\phi = 85\%$. As shown in the inset, for averaging radii smaller than approximately $120\ \mu\text{m}$ the slip coefficient becomes negative, indicating a breakdown of its definition, which implicitly assumes positive values. At sub-REV scales, the slip velocity falls below the Darcy velocity, leading to unphysical estimates of the coefficient. Only when the averaging radius approaches the REV scale does the slip velocity increase, yielding physically meaningful slip coefficients. This result demonstrates that slip coefficients must be evaluated exclusively at the REV scale, using the smallest representative averaging volume.

Finally, the bottom-right panel reports the results for the disordered porous medium. As in the ordered case at $\phi = 55\%$, the Beavers–Joseph and Saffman coefficients converge as the averaging volume approaches the REV scale, whereas at local scales, their discrepancy reaches approximately 70%. This panel also highlights the effect of the averaging kernel shape. For the same filter size, the Beavers–Joseph coefficient obtained with a pyramidal kernel is systematically larger than that obtained with a circular top-hat filter, indicating that both kernel size and shape influence the slip coefficient. These results expose a key limitation in comparisons of slip coefficients reported in the literature, which often arise from differences in the averaging procedure. To enable meaningful comparisons, both the kernel shape and its characteristic size must be explicitly specified. Accordingly, slip coefficients should be uniquely identified as $\alpha_{r_0}^{\text{kernel}}$ (e.g. $\alpha_{150}^{\text{prmd}}$, $\alpha_{200}^{\text{prmd}}$, $\alpha_{250}^{\text{prmd}}$, or α_{200}^{th}), providing an unambiguous definition of the scale and averaging method used.

5. CONCLUSIONS

This study provides experimental evidence that the interfacial slip coefficients used for coupling free flow and porous media depend on the observation scale, and therefore should not be regarded as intrinsic material properties. Using high-resolution μPIV in ordered and disordered porous media, pore-scale velocity fields were reconstructed and systematically upscaled to the REV scale. The interfacial slip velocity is primarily set by local pore geometry near the interface, and is comparable between ordered and disordered media at a given porosity, whereas the Darcy velocity within the porous matrix is strongly reduced by structural disorder through increased tortuosity and altered connectivity.

Critically, both the Beavers–Joseph and Saffman coefficients depend on the averaging procedure used to define the mesoscopic fields. While alternative reasonable averaging kernels may change the numerical value of the extracted coefficient, they do not remove the observed scale dependence. Rather, they define a different effective closure parameter associated with that particular averaging framework. Their values vary with averaging radius r_0 and kernel shape, and the two coefficients converge only at REV-scale averaging. At local or sub-REV scales, discrepancies can exceed 100% and, in highly permeable ordered porous media, insufficient averaging can even produce non-physical negative values. These results explain part of the scatter in published slip coefficients and underline the need for standardized, geometry-aware protocols. The present findings were obtained under low-Reynolds-number, quasi-two-dimensional Hele–Shaw flow conditions. Whether the same scale dependence persists in fully three-dimensional porous

structures and at higher Reynolds numbers remains an open question and should be addressed in future studies. Accordingly, slip coefficients should be interpreted as effective mesoscopic closure parameters associated with a specific averaging framework rather than as universal intrinsic material properties, and should therefore be reported as $\alpha_{r_0}^{\text{kernel}}$, together with the full averaging specification, to ensure reproducibility and meaningful cross-study comparisons.

STATEMENTS AND DECLARATIONS

Data, Code & Protocol Availability

The data, code, and protocols supporting the findings of this study are available from the corresponding author upon reasonable request.

Author Contributions

M. Del Mastro: Conceptualization (equal), Data Curation, Formal analysis, Investigation (equal), Methodology (equal), Writing - original draft (equal), Writing - review & editing (equal); **A. Terzis:** Conceptualization (equal), Funding acquisition (equal), Investigation (equal), Methodology (equal), Writing - Review & Editing (equal).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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