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Supplementary Material

HYBRID MIXTURE THEORY-BASED MODELING OF UNSATURATED TRANSPORT IN A DEFORMING POROUS FOOD MATRIX DURING FRYING

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APPENDIX: CONVERSION FROM EULERIAN TO LAGRANGIAN COORDINATES

The relation between an undeformed volume element, dV, and its deformed counterpart, dv, is given by **Equation A1** (1), where j (jacobian) is the determinant of the deformation gradient, $\boldsymbol{F}_{kK}$, which in cylindrical coordinates, $\bar{\boldsymbol{r}}(r,\theta,z)$, is expressed using indicial notation as **Equation A2**:

$$dv = jdV \tag{A1}$$

$$\boldsymbol{F}_{kK} = \bar{\boldsymbol{r}}_{k,K} \tag{A2}$$

The expression for the volume element (dv or dV) in cylindrical coordinates can be found in the following manner:

$$dv = dxdydz = \left|\frac{\partial(x,y,z)}{\partial(r,\theta,z)}\right| drd\theta dz \tag{A3}$$

$$\Rightarrow dv = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{vmatrix} drd\theta dz \tag{A4}$$

Using the relation between cartesian and cylindrical coordinates ($x = rcos\theta, y = rsin\theta, z = z$) in **Equation A4** gives:

$$dv = rdrd\theta dz \tag{A5}$$

We can rewrite **Equation A1** as:

$$rdrd\theta dz = j\mathrm{RdRd}\Theta dZ \tag{A6}$$

Further, it is assumed that the shape of the fried samples remains geometrically similar to their initial state, and there are no shape changes in the angular (θ) direction. Consequently, we have **Equation A7** and **A8**:

$$\frac{r}{\mathrm{R}} = \frac{dr}{d\mathrm{R}} = \frac{dz}{dZ} = \xi \tag{A7}$$

$$d\theta = d\Theta \tag{A8}$$

Using **Equations A7** and **A8** in **Equation A6**, we get,

$$j = \xi^3 \tag{A9}$$

For the solid phase, we can write (equating the mass of solids at $t = 0$ and $t = t$) (2).

$$j\varepsilon^s \rho^s = \varepsilon_0^s \rho_0^s \tag{A10}$$

where the subscript '0' stands for the initial state, i.e., the initial volume fraction (ε_0^s) and density (ρ_0^s) of the solid phase. We assume the solid phase to be incompressible ($\rho^s = \rho_0^s$), and can rewrite **Equation A10** as,

$$j = \frac{\varepsilon_0^s}{\varepsilon^s} = \frac{1 - \phi_0}{1 - \phi} \tag{A11}$$

where ϕ_0 represents the initial porosity of the matrix.

The cylindrical gradient operator in Eulerian coordinates, ∇_E, can be converted to the gradient operator in Lagrangian coordinates (∇_L) in the following manner:

$$\nabla_E \equiv \frac{\partial}{\partial r}\hat{r} + \frac{\partial}{\partial z}\hat{z} \tag{A12}$$

Using the chain rule and **Equation A7**, we can write,

$$\nabla_E = \frac{\partial \mathrm{R}}{\partial r}\frac{\partial}{\partial \mathrm{R}}\widehat{\mathrm{R}} + \frac{\partial Z}{\partial z}\frac{\partial}{\partial Z}\hat{Z} = \frac{1}{\xi}\left(\frac{\partial}{\partial \mathrm{R}}\widehat{\mathrm{R}} + \frac{\partial}{\partial Z}\hat{Z}\right) \tag{A13}$$

$$\Rightarrow \nabla_E = \frac{1}{\xi}(\nabla_L) = \left(\frac{1 - \phi}{1 - \phi_0}\right)^{\frac{1}{3}} \nabla_L \tag{A14}$$

Equation A14 is used to replace ∇_E in **Equations 16-19** (in main text) with ∇_L.

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