InterPore JOURNAL

INHERENT ERRORS IN CURRENT CORE-FLOODING RELATIVE PERMEABILITY DATA FOR MODELLING UNDERGROUND HYDROGEN STORAGE

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APPENDIX A

The new methodology for deriving relative permeability (RP) functions recommended in this paper has been validated directly against large scale CT imaged displacements (2). See **Figure S1** for an example of the simulation match to visualized experiment. One can see that there is a good qualitative match between the experimental finger pattern and that obtained via a simulation performed using the maximum mobility method. A close match was also found for the oil recovery, water cut, and pressure drop. Full details of the experiment and simulation are given in Beteta et al (2).

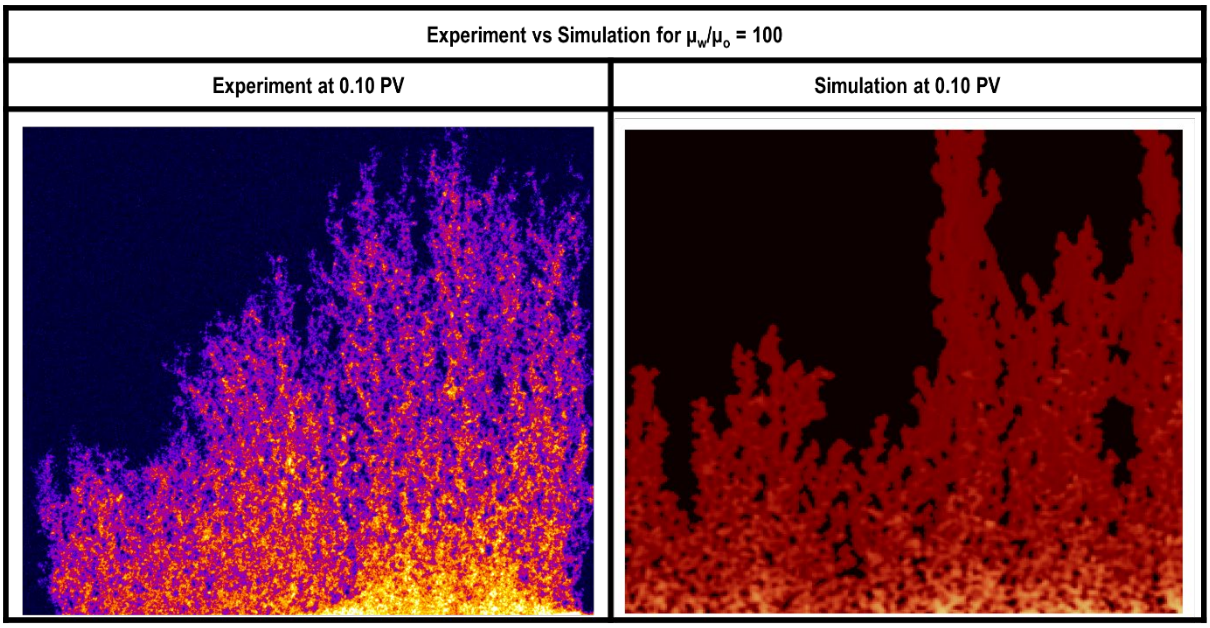

Figure S1: Experiment vs Simulation Match for Water Displacing Oil at an Adverse Viscosity Ratio of μ_w/μ_o = 100. Injection from Bottom to Top. Simulation Performed using Methodology from Sorbie et al (5). Full Details in Beteta et al (2).

APPENDIX B

Is there a critical or threshold capillary number where we go from capillary dominated to viscous dominated flow?

This question arose in the review of this paper: Capillary number – defined as $N_{Ca} = \mu \cdot v/\sigma$ – has been found to be a useful quantity to discuss certain aspects of core flooding, capillary desaturation (in cores) etc. Indeed, this capillary number was quoted in the text to give some approximate idea of "where" some of the core floods were located in "core analysis" terms. However, this capillary number, is a woefully inadequate quantity to quantitatively describe the rescaling of systems in terms of the viscous/capillary force balance. The simple version above does not even have a *length scale*. See Guo et al (3). Therefore, asking about where the "threshold capillary number" is for the transition to viscous dominated flow, is the wrong question.

To understand the rescaling of the results, we apply the Rapoport (4) scaling groups as explained in detail in Beteta et al (1), and **Figure S2** below is from this paper. Two of the Rapoport scaling groups for viscous/capillary ratio (C_{VC1}) and the "shape group" (C_{S1}) are also required to rescale the (2D) system, as given below.

$$C_{VC1} = \frac{q.\Delta x.\mu_o}{k_x.\Delta y.\Delta z\left(\frac{dP_c}{dS_w}\right)} = \frac{V.\Delta x.\mu_o}{k_x\left(\frac{dP_c}{dS_w}\right)}$$

$$C_{S1} = \frac{k_x \Delta y^2}{k_y \Delta x^2}$$

In **Figure S2**, System A is the lab scale system (with both viscous dominated RP functions + capillary pressure, P_c; see paper for actual functions (1)). No fingering is observed in A at a fixed flow rate (v) since capillarity is "important". However, as the system size increases (A → B → C→ D) while maintaining the same overall flow rate (v), RP and P_c, fingering becomes increasingly evident. The field-scale system D is virtually identical to lab-scale system A', which has the same structure on a smaller scale and the same RP functions but P_c =0. Note that the capillary number is *identical* for all of these cases in **Figure S2 (A-D)**, so it this number gives no information on whether the large-scale flow is dominated by capillary or viscous forces.

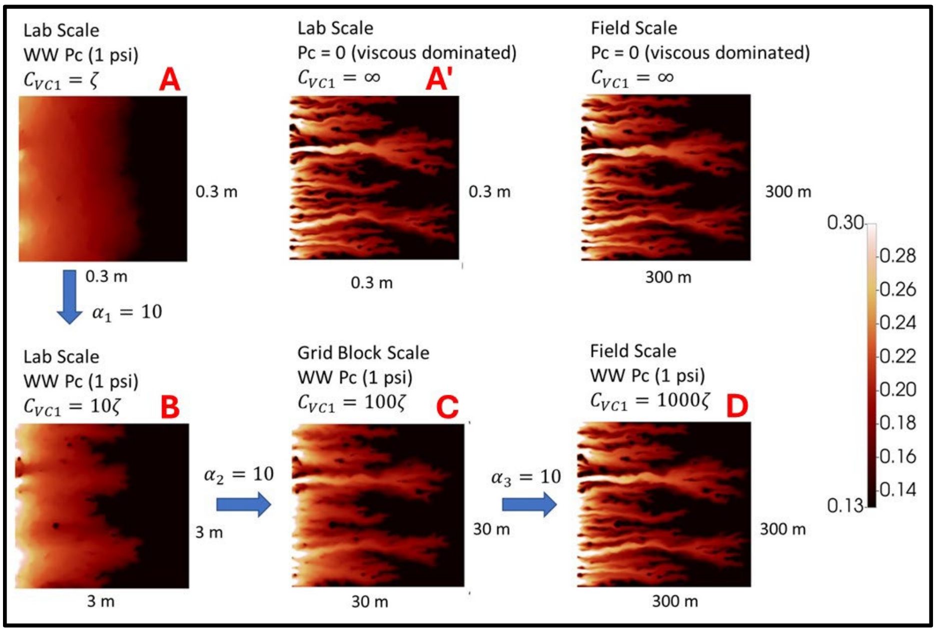

Figure S2: Water saturation finger patterns with and without water-wet (WW) P_c across a range of system size inflation factors (a_1, a_2, a_3, etc.); after Beteta et al (1).

In the actual 2D system shown in **Figure S2**, the length scale where it appears to be viscous dominated (i.e. almost identical to the zero P_c case in A') turns out to be in the range 3m < L < 30m or so. However, we have run examples where it can be ~0.1m to ~100m – it depends on the magnitude of the P_c and the other variables *v*, and size and permeabilities. Even the value of C_{VC1} does not give you an *absolute* "critical or threshold capillary number". However, using this scaling approach *guarantees* that if the RP functions are "fingering RP functions", then the fingering *must* emerge at some length scale, which we can only establish by calculation. The corollary to this is that, if with P_c = 0 in a mildly heterogeneous field, then no fingering is observed, then it will *never* emerge no matter what you do, for a "similar" system of any size. And here we use "similar" in the strict geometrical sense.

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