

EFFECT OF FISSURE DRAINAGE ON DISPERSIVE MIXING IN POROUS MEDIA GRAVITY CURRENTS: AN EXPERIMENTAL INVESTIGATION

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ABSTRACT

This Supplementary Material provides additional details supporting the main paper, including a comprehensive summary of all laboratory experiments conducted in this study and a brief review of the derivation of the theoretical model originally presented in Sheikhi et al. (4).



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A. LIST OF EXPERIMENTS

A detailed list summarizing all of the laboratory experiments conducted over the course of this study is provided in the table that follows.

Table S1: Experimental parameters, organized by experiment identifier (Expt. ID). Here, θ refers to the tank inclination angle, $\mathcal{N}$ refers to the number of open fissure holes, At refers to the Atwood number (defined by **Eq. 6** in the main text), z_n^* refers to the non-dimensional nozzle elevation and ‘Duration’ refers to the non-dimensional time needed for $x_{nose,disp}^* = 4x_f$. A dash in the final column signifies that the dispersed phase became arrested before $x_{nose,disp}^*$ reached $4x_f$.

Expt. ID	θ (°)	$\mathcal{N}$	At	$z_n^* = z_n/H$	Duration
E1	5	0	0.01	0	2.1
E2	5	0	0.02	0	2.4
E3	5	0	0.01	0.1989	2.4
E4	5	0	0.02	0.1469	2.6
E5	5	2	0.01	0	2.6
E6	5	2	0.02	0	2.9
E7	5	2	0.01	0.1989	3.0
E8	5	2	0.02	0.1469	2.9
E9	5	4	0.01	0	3.2
E10	5	4	0.02	0	3.7
E11	5	4	0.01	0.1989	3.5
E12	5	4	0.02	0.1469	3.7
E13	5	6	0.01	0	3.9
E14	5	6	0.02	0	4.0

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Table S1 – continued from previous page

Expt. ID	θ (°)	$\mathcal{N}$	At	$z_n^* = z_n/H$	Duration
E15	5	6	0.01	0.1989	3.7
E16	5	6	0.02	0.1469	4.2
E17	5	8	0.01	0	4.0
E18	5	8	0.02	0	4.3
E19	5	8	0.01	0.1989	3.7
E20	5	8	0.02	0.1469	4.4
E21	5	10	0.01	0	4.2
E22	5	10	0.02	0	4.4
E23	5	10	0.01	0.1989	3.7
E24	5	10	0.02	0.1469	5.1
E25	2.5	2	0.01	0	2.8
E26	2.5	2	0.02	0	3.3
E27	2.5	2	0.01	0.1989	3.2
E28	2.5	2	0.02	0.1469	3.3
E29	0	0	0.01	0	3.0
E30	0	0	0.02	0	3.2
E31	0	0	0.01	0.1989	2.6
E32	0	0	0.02	0.1469	4.2
E33	-2.5	2	0.01	0	3.9
E34	-2.5	2	0.02	0	4.8
E35	-2.5	2	0.01	0.1989	3.8
E36	-2.5	2	0.02	0.1469	4.3
E37	-5	0	0.01	0	3.3
E38	-5	0	0.02	0	3.9
E39	-5	0	0.01	0.1989	3.5
E40	-5	0	0.02	0.1469	4.3
E41	-5	2	0.01	0	4.4
E42	-5	2	0.02	0	6.3
E43	-5	2	0.01	0.1989	5.9
E44	-5	2	0.02	0.1469	7.4
E45	-5	4	0.01	0	14.3
E46	-5	4	0.02	0	-
E47	-5	4	0.01	0.1989	7.6
E48	-5	4	0.02	0.1469	-

B. THEORETICAL MODEL SUMMARY

A comprehensive derivation of the theoretical model is presented in Sheikhi et al. (4); only a brief review is provided here.

We assume a two-dimensional gravity current flow as depicted schematically in **Figure 2** (main text) where the bulk (density ρ_s , height h_{bulk}), dispersed (density ρ_{disp} , height $h_{\text{disp}} - h_{\text{bulk}}$) and ambient (density ρ_a) phases are all miscible (and incompressible). With reference to the coordinate system indicated in **Figure 2** (main text), mass continuity as applied to the bulk phase requires that

$$\phi \frac{\partial h_{\text{bulk}}}{\partial t} + \frac{\partial}{\partial x}(u_{\text{bulk}} h_{\text{bulk}}) = -w_{e1} - w_d F(x, x_f, \xi) \quad (\text{S1})$$

where ξ is the width of the (line) fissure and $F(x, \xi)$ is a boxcar function centered on this fissure. In **Equation S1**, reference is made to three Darcy velocities; u_{bulk} is the z -independent speed of the bulk phase, w_d accounts for fissure drainage and w_{e1} accounts for entrainment from the bulk to the dispersed phase. For simplicity, we assume a linear entrainment relationship such that $w_{e1} = \epsilon u_{\text{bulk}}$, where ϵ is an entrainment coefficient. In the dispersed phase, meanwhile, mass considerations yield:

$$\phi \frac{\partial (h_{\text{disp}} - h_{\text{bulk}})}{\partial t} + \frac{\partial}{\partial x} \int_{h_{\text{bulk}}}^{h_{\text{disp}}} u_{\text{disp}} dz = w_{e1} + w_{e2} \quad (\text{S2})$$

Here, $u_{\text{disp}} \neq u_{\text{bulk}}$ is the speed of the dispersed phase and $w_{e2} = \epsilon u_{\text{disp}}$ accounts for entrainment from the ambient to the dispersed phase. Application of **Equation S1** in **Equation S2** shows that:

$$\phi \frac{\partial h_{\text{disp}}}{\partial t} + \frac{\partial}{\partial x} [u_{\text{disp}} (h_{\text{disp}} - h_{\text{bulk}})] = -\frac{\partial}{\partial x} (u_{\text{bulk}} h_{\text{bulk}}) + w_{e2} - w_d F(x, x_f, \xi) \quad (\text{S3})$$

Whereas the solute concentration in the bulk phase, c_s , matches that of the source, the solute concentration, c_{disp} , in the dispersed phase must be computed from the following conservation equation:

$$\phi \frac{\partial}{\partial t} \int_{h_{\text{bulk}}}^{h_{\text{disp}}} c_{\text{disp}} dz + \frac{\partial}{\partial x} \int_{h_{\text{bulk}}}^{h_{\text{disp}}} u_{\text{disp}} c_{\text{disp}} dz = w_{e1} c_s \quad (\text{S4})$$

This last result can be simplified to:

$$\phi \frac{\partial b_{\text{disp}}}{\partial t} + \frac{\partial}{\partial x} (u_{\text{disp}} b_{\text{disp}}) = w_{e1} c_s \quad (\text{S5})$$

by defining $b_{\text{disp}} = \bar{c}_{\text{disp}} (h_{\text{disp}} - h_{\text{bulk}})$ where $\bar{c}_{\text{disp}}$ is the z -averaged solute concentration in the dispersed phase. **Equations S1**, **S3** and **S5** can be solved for h_{bulk} , h_{disp} and b_{disp} , provided expressions are available for the Darcy velocities u_{bulk} , u_{disp} and w_d . The expressions in question derive from Darcy's law where we assume (i) a hydrostatic pressure gradient, and, (ii) a linear equation of state. It can be shown that:

$$u_{\text{bulk}}(x, t) = -\frac{kg\beta}{\nu} \left[\frac{\partial b_{\text{disp}}}{\partial x} \cos\theta + c_s \left(\frac{\partial h_{\text{bulk}}}{\partial x} \cos\theta - \sin\theta \right) \right] \quad (\text{S6})$$

$$u_{\text{disp}}(x, t) = -\frac{kg\beta}{\nu} \left[\frac{\partial}{\partial x} \left(\frac{b_{\text{disp}} h_{\text{disp}}}{h_{\text{disp}} - h_{\text{bulk}}} \right) \cos \theta - \frac{b_{\text{disp}}}{h_{\text{disp}} - h_{\text{bulk}}} \sin \theta \right] \quad (\text{S7})$$

and

$$w_d(x, t) = \frac{k_f g \beta}{\nu} \left(\frac{c_s h_{\text{bulk}} + b_{\text{disp}}}{l} + c_s \right) \cos \theta \quad (\text{S8})$$

Here, g is gravitational acceleration, β is the solute contraction coefficient, and ν is the kinematic viscosity. Meanwhile k_f and l are the permeability and depth of the fissure.

Important to acknowledge in the context of **Table 1** (main text) is that the k_f of **Equation S8** is used as a fitting parameter and is chosen to minimize the overall error between $\bar{B}_{\text{disp,thy}}^*$ and $\bar{B}_{\text{disp,expt}}^*$. Thus do we select $k_f = 5.0k$ when $At = 0.01$ and $k_f = 2.5k$ when $At = 0.02$. The justification for making k_f a function of At comes from figure 7 of Roes et al. (3), which confirms that, similar to the discharge coefficients for tank flow, fissure permeabilities depend on the density of the discharging fluid. The mechanistic rationalization comes from the study of Holford & Hunt (2), who surmised that density effects may cause a contraction of the discharging flow over and above that associated with inertia.

In this respect, note that **(i)** similar observations have been made in the context of porous media flow, i.e. Bolster et al. (1) ascertained that the effective permeability of a heterogeneous medium may be influenced by the fluid density, and **(ii)** fitting the value of k_f allows us to apply a purely rectilinear theoretical model to experimental data in which fissure drainage occurs via up to $\mathcal{N} = 10$ discrete circular openings.

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