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Supplementary Material

MODELING DESICATION CRACKS IN OPALINUS CLAY AT FIELD SCALE WITH THE PHASE-FIELD APPROACH

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APPENDIX A: STRAIN ENERGY DECOMPOSITION

For isotropic linear elasticity, the strain energy density is given by

$$\Psi = \frac{1}{2}\boldsymbol{\varepsilon}:\mathbb{C}:\boldsymbol{\varepsilon}$$

$$= \frac{\lambda}{2}\mathrm{tr}(\boldsymbol{\varepsilon})^2 + G\mathrm{tr}(\boldsymbol{\varepsilon}:\boldsymbol{\varepsilon})$$

and the effective stress and the tangential stiffness tensor are given as

$$\boldsymbol{\sigma}_{\mathrm{eff}} = \frac{\partial\Psi}{\partial\boldsymbol{\varepsilon}} = \lambda\mathrm{tr}(\boldsymbol{\varepsilon})\mathrm{I} + 2G\boldsymbol{\varepsilon}$$

$$\mathbb{C} = \frac{\partial\boldsymbol{\sigma}_{\mathrm{eff}}}{\partial\boldsymbol{\varepsilon}} = \mathrm{K}\mathbf{I}\otimes\mathbf{I} + 2\mathrm{G}\left(\mathbb{I} - \frac{1}{3}\mathbf{I}\otimes\mathbf{I}\right)$$

The elastic constants are computed with

$$\lambda = \frac{Ev}{(1-v)\cdot(1-2v)}$$

and

$$G = \frac{E}{2(1+v)}$$

The elastic modulus, Poisson's ratio and shear modulus are represented by E, v and G respectively. To further simplify the notation, the bulk modulus $K = \lambda + 2/3G$ is used.

If we do not differentiate between active Ψ_+ and inactive Ψ_- parts of the strain energy

$$\Psi_+ = \Psi \text{ and } \Psi_- = 0$$

For this reason, any strain energy evolution is considered "active" and contributes to stiffness degradation $\mathbb{C}(d) = g(d)\mathbb{C}$. In the volumetric-deviatoric split [2], the strain energy is decomposed as

$$\Psi_{+} = \frac{K}{2}\langle \mathrm{tr}(\boldsymbol{\varepsilon})\rangle_{+}^{2} + G\boldsymbol{\varepsilon}^{\mathrm{dev}} : \boldsymbol{\varepsilon}^{\mathrm{dev}}$$

and

$$\Psi_{-} = \frac{K}{2}\langle \mathrm{tr}(\boldsymbol{\varepsilon})\rangle_{-}^{2}$$

where $\langle\cdot\rangle$ denotes the Macaulay brackets defined as $\langle a\rangle_{\pm} = (a \pm |a|)\ /\ 2$ and $\boldsymbol{\varepsilon}^{\mathrm{dev}}$ is the deviatoric part of the strain defined as:

$$\boldsymbol{\varepsilon}^{\mathrm{dev}} := \boldsymbol{\varepsilon} - \frac{1}{3}\mathrm{tr}(\boldsymbol{\varepsilon})\mathrm{I}$$

Accordingly, the stresses are given as

$$\boldsymbol{\sigma}_{\mathrm{eff}+} = \frac{\partial\Psi_{+}}{\partial\boldsymbol{\varepsilon}} = K\langle \mathrm{tr}(\boldsymbol{\varepsilon})\rangle_{+}\mathbf{I} + 2G\boldsymbol{\varepsilon}^{\mathrm{dev}}$$

and

$$\boldsymbol{\sigma}_{\mathrm{eff}-} = \frac{\partial\Psi_{-}}{\partial\boldsymbol{\varepsilon}} = K\langle \mathrm{tr}(\boldsymbol{\varepsilon})\rangle_{-}\mathbf{I}$$

The stiffness tensor ($\mathbb{C}(d) = g(d)\mathbb{C}_{+} + \mathbb{C}_{-}$) is

$$\mathbb{C}_{+} = \frac{\partial\boldsymbol{\sigma}_{+}}{\partial\boldsymbol{\varepsilon}} = H\big(\mathrm{tr}(\boldsymbol{\varepsilon})\big)K\mathbf{I}\otimes\mathbf{I} + 2G\left(\mathbb{I} - \frac{1}{3}\mathbf{I}\otimes\mathbf{I}\right)$$

$$\mathbb{C}_{-} = \frac{\partial\boldsymbol{\sigma}_{-}}{\partial\boldsymbol{\varepsilon}} = H\big(-\mathrm{tr}(\boldsymbol{\varepsilon})\big)K\mathbf{I}\otimes\mathbf{I}$$

where $H(\cdot)$ is the Heaviside step function defined in this study as:

$$H(x) := \begin{cases} 0, x < 0 \\ 1, x \geq 0 \end{cases}$$

Correction from May 3, 2024:

Please note that the pdf for this article was updated on May 3, 2024. Following online publication, the authors noticed some errors regarding the use of italics and bolding in several equations.